CONCERNING THE RELATION BETWEEN THE DURATION, INTENSITY, AND THE PERIODICITY OF RAINFALL.

By Prof. PETER PHILIPOVITCH GORBATCHEV.

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According to theoretical calculation, a definite quantity of water vapor brought by cyclonic air currents from the point of evaporation to the point of observation in form of a cloud, with a degree of humidity just sufficient to begin condensation, can produce precipitation, the general amount of which, h and the duration, t, can vary with the length of the cloud and the velocity of the air currents in which it floats. But for all this equally possible rain, that is, for rain precipitated from the same cloud, there must exist a definite relation between the average intensity of precipitation during the time of the rainfall, $i=h/t$, and the duration of that rainfall, t, that is

$$i_1\sqrt{t_1}=i_2\sqrt{t_2}=i_n\sqrt{t_n}=\Delta$$

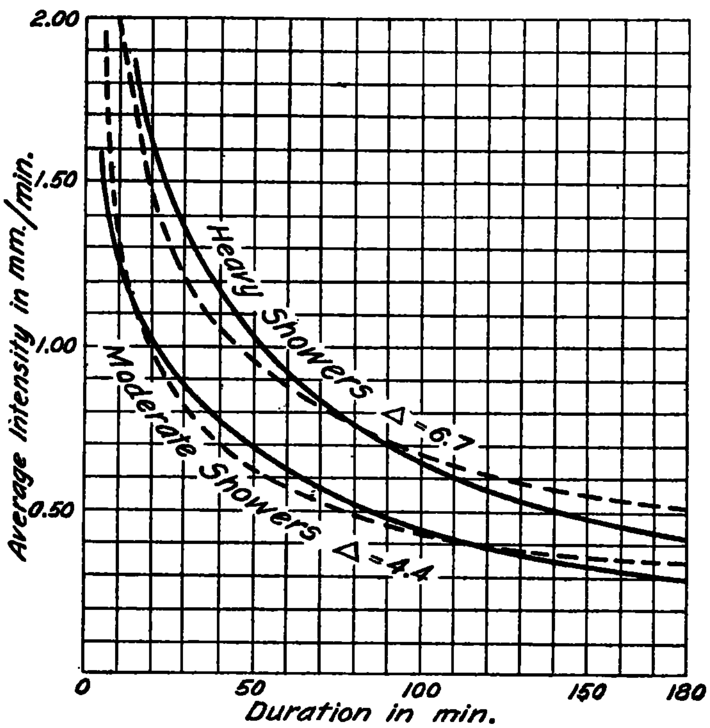

FIG. 1.—Curves of intensity ($i=\Delta/\sqrt{t}$) of showers in Middle Germany (after Hellmann).

Here the quantity Δ depends on the one hand, on the quantity of the drifted rain material in the cloud, and on the other hand on the angle of ascending cyclonic motion of the cloud, which is a result of topographical conditions, but it does not depend on the velocity of the storm nor on its length. Therefore, the quantity Δ for every separate cloud passing above a given place is a constant quantity and is characteristic of it and, therefore, of the whole series of rains equally possible from this cloud.

If the quantity of the water vapor in the cloud happens to be the maximum that is ever possible in the given period, owing to the extreme evaporation as a result of insolation and other favorable conditions, then surely all the equally possible precipitation from such a cloud will be stronger than from every other cloud. Consequently, the values of the intensities of such precipitations for corresponding periods will be the limit for the given place during the time considered, the same as the quantity Δ itself. If the quantity Δ is known, it is possible to construct a theoretical curve for limiting intensitities after the equation, $i=\Delta/\sqrt{t}$ for different durations.

If one compares the results of investigations of different authors concerning the limiting intensities for different places, obtained by selecting from meteorological records for a given period of time the observed intensities for each duration, it will appear that in all cases, those limiting intensities, expressed by empirical formulae or curves or tables, come always very near and sometimes coincide with theoretical curves constructed after the equation $i=\Delta/\sqrt{t}$ by corresponding average value of Δ. So, for instance, expressing h and i in mm. and t in minutes, the curves of the heavy showers for North Germany, according to the formula of Hellmann, will correspond to the value of $\Delta=6.7$ and the curve of moderate showers will be $\Delta=4.4$. (Fig. 1.) The curve of heaviest rains for the United States of America, after A. J. Henry, corresponds to $\Delta=12.4$, and for the heaviest showers in the southwest of Russia $\Delta=11.7$ (after Dolgov). According to A. Follwell the curves for moderate showers for North America will be as follows: Central States—$\Delta=5.2$; New England, $\Delta=3.5$ (Fig. 2); and for South Atlantic States, —$\Delta=5.7$. Berlin has $\Delta=3.0$ (according to Frühling for the space of a year) and Darmstadt —$\Delta=2.3$ (according to Heyd for the space of a year).

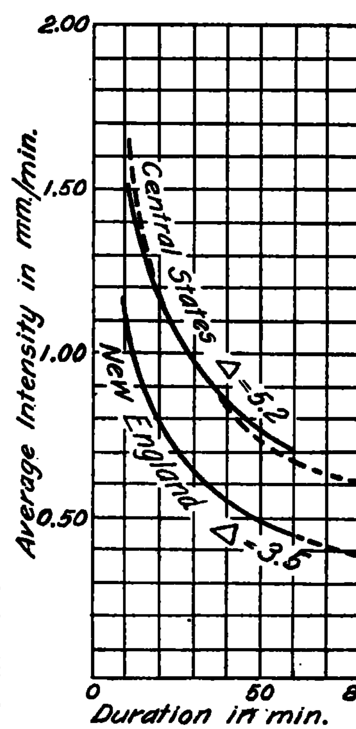

FIG. 2.—Curves of intensity of moderate showers in the United States (after Follwell).

According to the condition $i=h/t$ one can present the expression for Δ in other terms: $\Delta=h/\sqrt{t}$ and $\Delta=\sqrt{hi}$. The last equation shows that the quantity Δ is simultaneously determined by the proportions of the two most important characteristics of the rain, that is, the quantity of the rain (the quantity h) and by the intensity of the rainfall (the intensity i). Therefore the quantity Δ may be called the "rain power" as similarly in electrical terminology the products of quantity of current (in amperes) to its intensity (in volts) gives the effect or the power of the motor.

This determination of the "rain power" is convenient while working with meteorological quantities and gives an exact criterion: (a) for the classification of rains; (b) for comparative estimation of different rains, and (c) for the verification of meteorological records if doubtful.

(4) According to the "rain powers," calculated by the author for more than 200 cases of separate rains and their practical estimation by descriptions conforming to established determinations for different degrees of precipitation, it is possible to divide the rains after their "power" into the following categories (if h and i be expressed in mm. and t in minutes).

Type of precipitation.	Characteristics.	Rain power, Δ.	
		From—	To—
1. Small rains	Of importance only for rural economics.	0.0	1.0
2. Ordinary rains	Occur yearly	1.1	3.0
3. Moderate showers	Produce streams in natural cavities	3.1	5.0
4. Heavy showers	Are mentioned in records as rare phenomena.	5.1	7.0
5. Extraordinary showers	Disastrous results	7.1 and higher.	

After the observation for the plains of Europe, one can take $\Delta=12$ as the upper limit for power of extraordinary showers (Bobersberg in Brandenburg, June 21, 1895, $\Delta=i\times\sqrt{t}=1.08\times\sqrt{120}=11.8$). The upper limits for the "power" of extraordinary showers in North American are approaching the same number (Merrill, Wis., July 24, 1912, $\Delta=0.62\sqrt{450}=13.2$). In mountainous countries the extraordinary showers reach the power of $\Delta=16$ (Nieder Marsberg in Provinz Westfahlen, August 6, 1897, $\Delta=2.29\sqrt{45}=15.4$). The showers annotated under Tropics have the power of $\Delta=26$ (Manila, the hurricane of October 19–20, 1882, $\Delta=6.77\sqrt{15}=28.9$.

(5) When comparing rains of different intensity and duration, one can get an exact estimate by comparison of their "rain powers." For instance, after the terrible catastrophe of Kukui Dam on Moscow Kursk railway, Russia, there was offered by the Ministry the formula of Köstlin based upon the rain power $\Delta=0.96\sqrt{10}=3.0$ for the calculation of capacity of pipes or railway culverts. But engineers find it insufficient in practice because there have been observed heavier showers which have been confirmed by special observations. But from the calculation of rain power for the Kukui shower, above mentioned ($\Delta=0.48\sqrt{240}=7.4$) one can see at once that it is to be placed among extraordinary showers and gives 25 times more precipitation than would follow from the formula of Köstlin.

To make such comparisons easier we give below tables of intensities and general amounts of precipitation with different durations for principal categories of rain power.

TABLE 1.—*Average intensities $i=\Delta/\sqrt{t}$ (in mm./min.).*

Categories of rain power $i=\Delta/\sqrt{t}$.	Duration of rainfall t =					
	15 minutes.	30 minutes.	45 minutes.	1 hour.	3 hours.	6 hours.
1.0	0.26	0.18	0.15	0.13	0.07	0.05
3.0	0.77	0.55	0.45	0.39	0.22	0.16
5.0	1.29	0.91	0.75	0.65	0.37	0.27
7.0	1.81	1.27	1.04	0.90	0.52	0.37
12.0	3.10	2.18	1.79	1.55	0.89	0.64
16.0	4.13	2.91	2.38	2.06	1.18	0.85
26.0	6.71	4.73	3.87	3.35	1.92	1.38

TABLE 2.—*General amount of rainfall $h=\Delta\sqrt{t}$.*

$h=\Delta\sqrt{t}$.	Duration of rainfall t =					
	15 minutes.	30 minutes.	45 minutes.	1 hour.	3 hours.	6 hours.
1.0	3.9	5.5	6.7	7.7	13.4	19.0
3.0	11.6	16.4	20.6	23.3	40.3	56.9
5.0	19.4	27.4	33.6	38.8	67.1	94.9
7.0	27.1	38.4	47.0	54.3	93.9	132.8
12.0	46.5	65.8	80.5	93.0	161.0	227.6
16.0	61.9	87.7	107.4	124.0	214.7	303.5
26.0	100.6	142.5	174.5	201.5	378.9	493.2

Since we know the largest possible "rain power" for different countries, we can verify the accuracy of separate observations. For instance, there had been doubts about the shower at Nagartava (prov. Cherson, Russia) July 9, 1921, which has been recorded as having $h=99$ mm., $t=30$ min. and $i=3.3$ mm./min. If one calculated the rain power according to it, it will be $\Delta=3.3\sqrt{30}=18.1$. As there are never on the plains of Europe showers with Δ more than 12.0, and in mountainous countries no more than $\Delta=16.0$, so there must be obviously a mistake in this record. And, indeed, there is an exact record in the detailed revision of Professor Klossovsky from which one can see that the rain lasted not for 30 minutes but for 4 hours and 30 minutes or 270 minutes, so that the intensity will be only $i=0.4$. Then the rain power will be $\Delta=0.4\sqrt{270}=6.6$ what [which] is a rather heavy but quite possible shower for the given country. There is also a shower quoted in the list of Professor Friedrich, that took place in Berlin, April 14, 1902, that was mentioned as having $h=143$ mm., $t=210$ min., and $i=1.18$ mm./min., which give the rain power $\Delta=1.18\sqrt{210}=17.1$, which is quite impossible for the Prussian plain. But according to a more careful examination of that remarkable shower, it appears from a list by Professor Hellmann and description of Professor Hergardt of the heaviest point of rainfall that exact numbers are as follows: $h=166$ mm., $t=345$ min., and $i=0.48$ mm./min. That corresponds to $\Delta=0.48\sqrt{345}=8.9$ which is quite probable for Berlin. It is very possible that some of the records quoted in literature about extraordinary showers (as for instance in Ardgis in Rumania, $h=205$ mm., $t=20$ min., and $\Delta=48.1$; in Campo, Calif., $h=292$ mm., $t=60$ min., and $\Delta=38.0$, and others) will require correction in order to make them conform to original records.

If one examines every shower separately it appears that the rainfall takes place somewhat unequally because of the irregularity of the air currents. But the departures of the values of the "rain power" for its separate parts from the average power of the rain for the whole time of its duration are not so great and do not surpass 10 per cent for every part of the rain from the beginning and no more than 15 per cent for the maximum taken from the middle of the rain, as one can see from the examination of a famous, extremely irregular, shower at Zurich, June 3, 1878, with $\Delta=4.0$ and with greatest power from the beginning of the rain with $\Delta=4.4$ and $\Delta=4.6$ for the maximum part in the middle of the rain. No more than such departures are observed among other rains, when comparing their average power with the largest in maximum parts. From this it follows that for determining rain powers one has always to calculate it as average power for the whole rain and not divide it into heavy and small parts; that for practical purposes, for technical calculations, for instance, one can assume without great error that the relation between the duration and the intensity of the rain is maintained in every part of it, and that the value of the "rain power" remains the same as for the whole rain, and for every part of the duration taking it from the beginning of the rainfall.

Up to this point the "rain power" Δ has been examined by the author independently of the length of the period of meteorological observations. But if one compares the limiting rain powers for the same point in different years, it will appear that the limits will be greater the longer the period of the observation. This is explained by the fact that the quantity Δ depends, as said above, upon the largest amount of drifted rain material; that is, from the largest evaporation and the strongest cyclones, which depends for itself upon the quantities of caloric energy emitted by the sun. Professor Brückner proved the periodicity of this phenomenon with the change of cold and warm years during the period of 30–35 years and related it to the sunspot maximum. But the comparison of spring floods of rivers shows that the greatest maxima repeat every two Brückner periods; that is, every 60 to 70 years; there are also some indications of two periods for the maxima of the solar spots, about 11 and about 60 years, that appear in the complete fluctuations in the curve of periodicity. Therefore one must wait also for the change of limiting "rain power," the period of about 35 or 70 years.

If one examines the number of the occurences exceeding some certain "rain power" by other heavier rains, a remarkable regularity is discovered; for example, if calculating the "rain powers" from their complete list for the given country during the period n years and tabulating them according to their powers beginning from the largest, we obtain K_1, K_2, and so on, cases for every observed power. If we take every power that exceeds the nearest next class of $\Delta_n - 0.1$ and add the number of cases, we have for any rain power $\Delta_n - 0.1$ the number of cases of exceeding $= K_1 + K_2 \ldots\ldots + K_n = \Sigma K$ for the whole period of n years and $\frac{\Sigma K}{n}$ for 1 year.

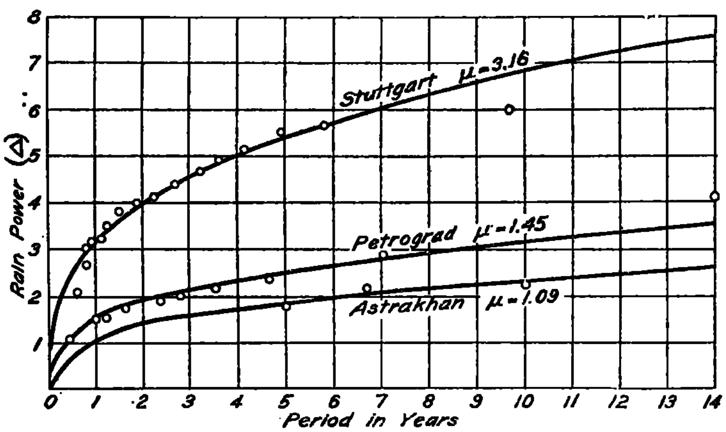

FIG. 3.—Curves of periodicity of "rain power" ($\Delta = \mu\sqrt[3]{p}$).

This quantity expresses the frequency of only one exceeding the given "rain power" in the course of one year $\frac{\Sigma K}{n} = \frac{1}{\rho}$ and the inverse quantity ρ will be the period for the only one exceeding the given "rain power." If we set down graphically the observed rain powers Δ as ordinates taking the periods ρ (the excedance interval) as abscissae the curves obtained for different countries will be all cubic parabolas, which means that there exists an equation $\Delta = \mu\sqrt[3]{\rho}$ where μ is a constant climatic factor for the country, expressing the correlation of all its climatic and geographical conditions. So, for instance, these climatic factors have proved to be for Hanover, $\mu = 2.90$; for Stuttgart, $\mu = 3.16$; for Petrograd, $\mu = 1.45$, and for Astrakhan $\mu = 1.09$. (Fig. 3.)

One may note that it follows from the corresponding equations that the climatic factor μ exceeded the "rain power" Δ no more than once a year, and generally the "rain power" Δ can be expressed as the theoretical intensity, corresponding to the precipitation of the whole amount of rain in the course of one minute. As an instance of the calculation of the climatic number μ there are given tables of its deduction for the city of Stuttgart from the records of 29 years' observation.

TABLE 3.—*Showers at Stuttgart, 1875–1903.*

Δ	9.3	7.4	6.0	5.7	5.6	5.3	5.0	4.0	4.5	4.3	4.1
Number of cases K	1	1	1	2	1	1	1	1	2	2	2

DEDUCTION OF CLIMATIC NUMBER, μ, FOR STUTTGART.

Observed rain power Δ.	ΣK	$\frac{m}{n} = \frac{1}{\rho}$	ρ	$\sqrt[3]{\rho}$	$\mu = \Delta / \sqrt[3]{\rho}$
More than 9.2	1	1/29	29	3.07	3.0
More than 7.3	2	2/29	14.50	2.44	3.0
More than 5.9	3	3/29	9.67	2.13	2.8
More than 5.6	5	5/29	5.80	1.80	3.1
More than 5.5	6	6/29	4.83	1.64	3.3
More than 5.2	7	7/29	4.15	1.61	3.2
More than 4.9	8	8/29	3.63	1.54	3.2
More than 4.8	9	9/29	3.22	1.48	3.2
More than 4.4	11	11/29	2.64	1.38	3.2
More than 4.2	13	13/29	2.23	1.31	3.2
More than 4.0	15	15/29	1.93	1.25	3.2

If one knows the climatic numbers for the given country, one can calculate from the equation $\Delta = \mu\sqrt[3]{\rho}$ for every period of time ρ such "rain power" Δ, as can be exceeded more than once for this period of time and that can be considered the limit for this period of time, and from which one can calculate from the equation $i = \Delta / \sqrt{t}$ the limiting possible intensities for every duration during this period of time. This is very important for technical calculation. So, for instance, in Germany one assumes for the projects of sewerage that the sewers are to overfill no more than once a year and on the main streets no more than once in two or three years. According to the author's opinion one should take as a period for the single overfilling no less than five years, and for the more dangerous cases one has to increase the period as follows, for instance: For deep valleys within the town, 20 years; for pipes and bridges for roadway, 35 years; and for railways, the largest period, 70 years.

Moreover, one can theoretically obtain from the equation $\Delta = \mu\sqrt[3]{\rho}$ some interesting deductions concerning the recurrence of rains of different powers, concerning the average yearly amount of precipitation, and lastly, concerning the largest possible rain powers which are justified by actual observations.

(10) Yearly recurrence of a group of any rain powers limited from Δ_0 to Δ_1 is expressed by the equation $\frac{1}{\alpha} = \mu^3 \frac{\Delta^3_0 - \Delta^3_1}{\Delta^3_0 \Delta^3_1}$, and inversely — α will be the number of years in the period of only one occurrence of any rain power from this group. Thus theoretically calculated numbers of cases of yearly occurrence nearly coincident with observed actual cases according to the rain records are proven in the table below for Stuttgart and Petrograd, but it is necessary to remark that there are not mentioned a great many small rains in the record of Stuttgart, and that heavy showers (with Δ more than 5.0) are very scarce in Petrograd and as, according to calculated yearly occurrence, $\alpha = 0.02$ they happen only once in 50 years, it is difficult to expect that they can be mentioned in a rain record of 19 years.

Yearly occurrence.	$\frac{1}{\alpha_0}=\mu^3_0\ \frac{\Delta^3_0-\Delta^3_1}{\Delta^3_0\ \Delta^3_1}$			
Categories of rains.	Stuttgart.		Petrograd.	
	Theoretical.	Actual.	Theoretical.	Actual.
Ordinary rains (1.0–3.0)	30.4	2.1	2.9	2.9
Moderate showers (3.0–5.0)	0.91	0.83	0.08	0.07
Heavy showers (5.0–7.0)	0.16	0.17	0.02	0.00

The average yearly amount of precipitation evidently consists of the sum of precipitation of all categories of rains corresponding to their yearly occurrence and one can express it theoretically by equation $H=S\sqrt{\mu^3}$. If we compute the quantity S for four towns with continental climate we obtain for Stuttgart $S=110$, for Hanover $S=116$; for Ekaterinoslav $S=120$, and for Astrakhan $S=131$. Therefore, one can take for the central region of Europe the middle number with sufficient accuracy and obtain the equation $H=120\sqrt{\mu^3}$. But for countries with humid sea climate the quantity S departs considerably from its average value (for instance, for Petrograd $S=271$). Probably there fall and are recorded very small rains which in a drier continental climate are evaporated without reaching the surface of the earth and therefore escape being recorded.

(12) The largest possible "rain power" for a given place will evidently be such a power as is not surpassed in the course of the complete cycle of periodicity of precipitation for ρ years, and consequently will be $\Delta=\mu\sqrt[3]{\rho}+0.1$. In case the exact climatic number of the country μ is unknown, it can be substituted by yearly average quantity of precipitation H (from the equation $H=120\sqrt{\mu^3}$), from which it becomes approximately $\Delta=0.041\sqrt[3]{H^2}\sqrt[3]{\rho}+0.1$.

The complete cycle of periodicity of precipitation makes up, according to Brückner, $\rho=35$ years, and then the maximal rain power will be $\Delta=0.13\sqrt[3]{H^2}+0.1$. But according to the opinion of the author the complete cycle of periodicity must be a double one—that is, $\rho=70$ years. The maximum possible "rain power" will be somewhat larger, namely, $\Delta=0.17\sqrt[3]{H^2}+0.1$.

For the verification of this formula there were calculated by the author the "powers" of all remarkable showers known to him, from which appeared to be that for the majority of actual observations the "rain powers" do not surpass the calculated value of their power for the period of $\rho=35$, although approaching it closely. That could be expected because the majority of exact meteorological reports seldom embraces a period of observations larger than for 30 to 40 years. Only in nine cases quoted below the actual maximum "rain power" was larger than the theoretical for the period of $\rho=35$ years, but did not reach the theoretical power calculated for the period of $\rho=70$ years.

Country.	Date.	Yearly H.	Theoretical.		Actual.
			Max. $\rho=35$	Δ $\rho=70$	Δ max.
		Feet.			
Budapest	June 26, 1875	435	7.6	9.9	8.5
Paris	Sept. 20, 1867	483	8.1	10.6	9.2
Treuenbrietzen (Brandenburg)	July 31, 1897	506	8.4	10.9	10.8
Vienna	July 21, 1912	566	9.0	11.7	9.9
Breslau	Aug. 6, 1858	585	9.2	11.9	10.1
Schwerin	May 11, 1870	614	9.5	12.4	11.4
Karlsruhe	June 23, 1885	723	10.6	13.8	13.0
Geneva	May 30, 1827	822	11.5	15.0	12.1
Nieder Marsberg	Aug. 6, 1897	975	12.9	16.8	15.4

TABLE 5.—*Rains at Stuttgart recorded during the period 1875–1903, according to Dr. Th. Heyd.*

[The intensity of precipitation in mm./min. is calculated by the author.]

Dates.	Intensity of precipitation (mm./min.).	Quantity of precipitation (e/sec. ha.).	Duration of precipitation (mins.).
1875—Aug. 31	1.20	200	7
1876—June 7	0.54	90	10
July 29	0.38	63	29
1877—June 20	0.67	112	34
June 21	1.20	200	60
July 14	0.44	74	15
July 18	0.35	58	10
1878—May 12	0.25	42	90
May 14	0.40	67	10
June 14	0.59	99	44
July 27	0.43	71	45
Aug. 7	0.34	56	17
Aug. 7	0.49	82	13
1879—Apr. 26	0.26	44	10
1880—May 14	0.62	104	10
June 11	0.55	91	35
July 1	0.59	99	16
Aug. 13	0.25	42	19
Sept. 8	0.28	47	10
Sept. 18	0.34	57	25
1881—July 9	0.76	124	12
July 16	0.22	37	75
1882—May 30	0.82	137	18½
May 30	0.73	121	12½
1883—May 8	0.52	86	20
July 10	0.85	142	22
July 10	1.04	174	15
July 23	2.50	417	3

Conforming to the above, the calculation for the rain power $\Delta=i\sqrt{t}$ and climatic numbers of country $\mu=\Delta\sqrt[3]{\rho}$ give a new method of working with meteorological observations, allowing us to establish some laws for phenomena of precipitation and obtain by theoretical means deductions confirmed by observations. Therefore, it is very desirable to include in all meteorological records information concerning the rain amounts h and duration t, also the calculated quantities of the rain power Δ. Summarizing the latter for many stations with regard to their duration, occurrence, and departure from their passage above some neighboring points of observations, and so on, and also studying the dependence of climatic factor for different countries μ from the location of points of observation relative to the mountain ranges and tracks of cyclones and their elevation above the sea level, it is possible to open new ways of exploration through the extremely abundant but hardly accessible virgin forest in which appears now the vast amount of meteorological observations concerning the rainfall of many thousands of stations in the whole world.

DISCUSSION.

By H. R. LEACH and R. E. HORTON.

[Voorheesville, N. Y., July 24, 1923.]

The suggestion that storms can be classified according to their "rain power" is worthy of further study. Once its true relation to other storm characteristics is established, and its frequency equation determined, most of the storm characteristics of a certain locality can be expressed in two or three simple equations, the constants of which may possibly hold for relatively large areas, as suggested in the paper.

The formula given for "rain power," $\Delta=i\sqrt{t}$ is not satisfactorily proven and is not in accordance with more recent intensity-duration formulas. The assumption that the power of a given storm is constant is not conclusively shown and it seems just as logical to assume that the power may suffer depletion as the storm progresses. The